

A Three Higgs Doublet Model for the Fermion Mass Hierarchy Problem

Wei Chao*

Department of Physics, University of Wisconsin-Madison, Madison, WI 53706, USA

Abstract

In this paper we propose an explanation to the Fermion mass hierarchy problem by fitting the type-II seesaw mechanism into the Higgs doublet sector, such that their vacuum expectation values are hierarchal. We extend the Standard Model with two extra Higgs doublets as well as a spontaneously broken $U_X(1)$ gauge symmetry. All fermion Yukawa couplings except that of top quark are of $\mathcal{O}(10^{-2})$ in our model. Constraints on the parameter space from Electroweak precision measurements are studied. Besides, the neutral component of the new fields, which are introduced to cancel the anomalies of the $U(1)_X$ gauge symmetry can be dark matter candidate. We investigate its signature in the dark matter direct detection.

*Electronic address: chaow@physics.wisc.edu

I. INTRODUCTION

In the Standard Model (SM) of particle interactions, charged fermions get masses through the spontaneously broken of the electroweak symmetry and the Higgs mechanism, while neutrinos are massless. At M_Z , the charged lepton masses and the current masses of quarks are given by [1]

$$\begin{aligned} m_e &\sim 0.51 \text{ MeV} & m_\mu &\sim 0.105 \text{ GeV} & m_\tau &\sim 1.7 \text{ GeV} \\ m_u &\sim 1 \text{ MeV} & m_c &\sim 1.3 \text{ GeV} & m_t &\sim 174 \text{ GeV} \\ m_d &\sim 5 \text{ MeV} & m_s &\sim 0.13 \text{ GeV} & m_b &\sim 4 \text{ GeV} , \end{aligned} \tag{1}$$

which shows an enormous hierarchy among the Yukawa couplings y_ψ . For example, we have $y_u/y_t \sim 10^{-5}$ for the quark sector.

For the neutrino sector, recent results from solar, atmosphere, accelerator and reactor neutrino oscillation experiments show that neutrinos have small but non-zero masses at the sub-eV scale and different lepton flavors are mixed. If neutrinos are Dirac particles, their masses may come from the Higgs mechanism, then we have $y_\nu/y_t \sim 10^{-12}$, which seems even unnatural. For the case neutrinos being Majorana particles, the most popular way to explain neutrino masses are the seesaw mechanism[2–4]. If we assume the Yukawa couplings between left-handed lepton doublet and right-handed neutrinos are of order 1, then we have $m_t/m_N \sim 10^{-12}$, which is also unnatural.

In this paper, we attempt to solve or explain the charged fermion and neutrino mass hierarchy problem in the three Higgs doublet model. There are already many excellent literatures focusing on this issue[5–17]. In our model, one Higgs doublet get its vacuum expectation value (VEV) in the same way as that of the SM Higgs boson, while the other two Higgs fields get their VEVs through the mechanism similar to type-II seesaw model¹, i.e., they get their VEVs through their mixings with the SM Higgs. Such that the VEVs can be normal hierarchal, which is guaranteed by the spontaneously broken $U(1)$ gauge symmetry. We set them to be $v_1 = 100 \text{ MeV}$, $v_2 = 10 \text{ GeV}$ and $v_3 = 173 \text{ GeV}$ in our paper. For each generation of charged fermions, there is one Higgs field responsible the origin of their masses. For the neutrino sector, there are only Yukawa couplings with the

¹ For similar ideas on the VEVs of Higgs doublet, see the private Higgs model[19], the two Higgs doublet model with softly breaking $U(1)$ symmetry[20] and [21–24] for neutrino masses.

first generation Higgs field. Such that Dirac neutrino mass matrix is naturally small without requiring small Yukawa coupling constants. Then active neutrinos may get small but non-zero masses through the TeV-scale seesaw mechanism [20]. We introduce some new fields to cancel anomalies of the $U(1)_X$ gauge symmetry, and the neutral component of them can be cold dark matter candidate. We will study its signatures in dark matter direct detection experiments.

The note is organized as follows: In section II we give a brief introduction to the model, including particle contents, Higgs potential and scalar mass spectrum. Section III is devoted to study the fermion masses. We investigate constraints on the model from Electroweak precision measurements and dark matter phenomenology in section IV and V. The last part is concluding and remarks.

II. THE MODEL

Fields	q_L^u	q_L^c	q_L^t	u_R	c_R	t_R	d_R	s_R	b_R	ℓ_L	e_R	μ_R	τ_R	ν_R^i	ψ_L^i	η_L^k	ξ_L^k	η_R^k	ξ_R^k	H_1	H_2	H_3	Φ
$U_X(1)$	1	-1	0	2	-2	0	0	0	0	0	-1	1	0	1	1	1	-1	0	0	1	-1	0	1

TABLE I: Particle contents and their quantum numbers under $U_X(1)$ gauge symmetry. $i = 1, 2, 3$ and $k = 1, \dots, 6$. $q_L^u = (u_L, d_L)^T$, $q_L^c = (c_L, s_L)^T$, $q_L^t = (t_L, b_L)^T$, ℓ_L denotes left-handed lepton doublets.

We extend the SM with three right-handed neutrinos, two extra Higgs doublet, one Higgs singlet as well as a flavor dependent $U(1)_X$ gauge symmetry. Six generation fermion singlets $\eta(\xi)$ with $U(1)_X$ hypecharge $(-)$ 1 as well as three generation fermion singlets ψ_L with $U(1)_X$ hypecharge 0 are introduced to cancel the anomalies. The particle contents and their representation under the $U(1)$ gauge symmetry are listed in table I. We apply the type-II seesaw mechanism to the Higgs doublet sector. The most general Higgs potential can be written as

$$\begin{aligned}
\mathcal{L}_{\text{Higgs}} = & +m_1^2 H_1^\dagger H_1 + m_2^2 H_2^\dagger H_2 - m_3^2 H_3^\dagger H_3 - m_0^2 \Phi^\dagger \Phi + \lambda_0 (\Phi^\dagger \Phi)^2 + \lambda_1 (H_1^\dagger H_1)^2 + \lambda_2 (H_2^\dagger H_2)^2 \\
& + \lambda_3 (H_3^\dagger H_3)^2 + \lambda_4 (H_1^\dagger H_1)(H_2^\dagger H_2) + \lambda_5 (H_1^\dagger H_1)(H_3^\dagger H_3) + \lambda_6 (H_2^\dagger H_2)(H_3^\dagger H_3) \\
& + \lambda_7 (H_1^\dagger H_2)(H_2^\dagger H_1) + \lambda_8 (H_1^\dagger H_3)(H_3^\dagger H_1) + \lambda_9 (H_2^\dagger H_3)(H_3^\dagger H_2) + \lambda_{10} (\Phi^\dagger \Phi)(H_1^\dagger H_1) \\
& + \lambda_{11} (\Phi^\dagger \Phi) H_2^\dagger H_2 + \lambda_{12} \Phi^\dagger \Phi H_3^\dagger H_3
\end{aligned}$$

$$+ \left(\lambda_{13} (H_3^\dagger H_1) (H_3^\dagger H_2) + \mu_1 \Phi H_3^\dagger H_1 + \mu_2 \Phi^\dagger H_3^\dagger H_2 + \text{h.c.} \right) . \quad (2)$$

It is obviously that H_1 and H_2 shall develop no VEVs without terms in the bracket of Eq. 2. The conditions for $\mathcal{L}_{\text{Higgs}}$ develops minimum involve four constraint equations. By assuming $\langle H \rangle = v_1/\sqrt{2}$, $\langle \eta \rangle = v_2/\sqrt{2}$, $\langle \varphi \rangle = v_3/\sqrt{2}$ and $\langle \Phi \rangle = v_4/\sqrt{2}$, we have

$$\begin{aligned} & +m_1^2 v_1 + \lambda_1 v_1^3 + \frac{1}{2} v_1 [(\lambda_4 + \lambda_7) v_2^2 + (\lambda_5 + \lambda_8) v_3^2 + \lambda_{10} v_4^2] + \frac{1}{2} \lambda_{13} v_2 v_3^2 + \mu_1 v_3 v_4 = 0 , \\ & +m_2^2 v_2 + \lambda_2 v_2^3 + \frac{1}{2} v_2 [(\lambda_4 + \lambda_7) v_1^2 + (\lambda_6 + \lambda_9) v_3^2 + \lambda_{11} v_4^2] + \frac{1}{2} \lambda_{13} v_2 v_3^2 + \mu_2 v_3 v_4 = 0 , \\ & -m_3^2 v_3 + \lambda_3 v_3^3 + \frac{1}{2} v_3 [(\lambda_5 + \lambda_8) v_1^2 + (\lambda_6 + \lambda_9) v_2^2 + \lambda_{12} v_4^2] + \lambda_{13} v_1 v_2 v_3 + \mu_1 v_1 v_4 + \mu_2 v_2 v_4 = 0 , \\ & -m_0^2 v_4 + \lambda_0 v_4^3 + \frac{1}{2} v_4 [\lambda_{10} v_1^2 + \lambda_{11} v_2^2 + \lambda_{12} v_3^2] + \mu_1 v_1 v_3 + \mu_2 v_2 v_3 = 0 . \end{aligned} \quad (3)$$

Let $m_i^2, \lambda_i > 0$, $\lambda_{13} = 0$ (for simplicity) and $|\mu_i| \ll m_i$, then we have

$$v_1 \approx \frac{\mu_1 v_3 v_4}{m_1^2} , \quad v_2 \approx \frac{\mu_2 v_3 v_4}{m_2^2} , \quad v_3^2 \approx \frac{m_3^2}{\lambda_3} , \quad v_4^2 \approx \frac{m_0^2}{\lambda_0} . \quad (4)$$

Notice that v_1 and v_2 are suppressed by their masses, which is quite similar to that in the type-II seesaw mechanism. So we can get relatively small v_1 and v_2 without conflicting with any electroweak precision measurements. By setting $m_1 \sim 10m_2$ and $\mu_1 \sim \mu_2$ we get the normal hierarchal VEVs for the Higgs sector. We set $\mathcal{O}(v_1) \sim 0.1$ GeV, $\mathcal{O}(v_2) \sim 1$ GeV and $\mathcal{O}(v_3) \sim 100$ GeV in our following calculation. In this way the fermion mass hierarchy problem will be fixed, as will be shown in the next section.

After all the symmetries are broken, there are four goldstone particles eaten by W^\pm, Z and Z' . The mass matrix for the CP-even Higgs bosons can be written as

$$M_{\text{even}}^2 \approx \begin{pmatrix} m_1^2 + v_1^2 \lambda_1 & \frac{1}{2} v_1 v_2 (\lambda_4 + \lambda_7) & \frac{1}{2} v_1 v_3 (\lambda_5 + \lambda_8) - \mu_1 v_4 & \frac{1}{2} v_1 v_4 \lambda_{10} - v_3 \mu_1 \\ * & m_2^2 + v_2^2 \lambda_2 & \frac{1}{2} v_2 v_3 (\lambda_6 + \lambda_9) - \mu_2 v_4 & \frac{1}{2} v_2 v_4 \lambda_{11} - v_3 \mu_2 \\ * & * & v_3^2 \lambda_3 & \frac{1}{2} v_3 v_4 \lambda_{12} - v_2 \mu_2 \\ * & * & * & v_4^2 \lambda_4 \end{pmatrix} \quad (5)$$

It can be block diagonalized and the mapping matrix can be written as

$$V \approx \begin{pmatrix} \mathcal{V}_1 & 0 \\ -\mathcal{T}^T \mathcal{Z}^{-1} & \mathcal{V}_2 \end{pmatrix} , \quad (6)$$

where \mathcal{V}_i is the 2×2 unitary matrix and the expressions of \mathcal{T} and \mathcal{Z} are listed in the appendix. The corresponding mass eigenvalues are then

$$M_1^2 \approx c^2 (m_1^2 + v_1^2 \lambda_1) + s^2 (m_2^2 + v_2^2 \lambda_2) + c s v_1 v_2 (\lambda_4 + \lambda_7) , \quad (7)$$

$$M_2^2 \approx s^2(m_1^2 + v_1^2\lambda_1) + c^2(m_2^2 + v_2^2\lambda_2) - cs v_1 v_2(\lambda_4 + \lambda_7), \quad (8)$$

$$M_3^2 \approx c'^2(v_3^2\lambda_3 - v_4^2\alpha) + s'^2(v_4^2\lambda_4 - v_3^2\alpha) - c's'v_3v_4(\lambda_{12} - 2\alpha), \quad (9)$$

$$M_4^2 \approx s'^2(v_3^2\lambda_3 - v_4^2\alpha) + c'^2(v_4^2\lambda_4 - v_3^2\alpha) + c's'v_3v_4(\lambda_{12} - 2\alpha), \quad (10)$$

where $\alpha = \mu^2 m_1^{-2} + \mu_2 m_2^{-2}$, $c^{(\prime)}, s^{(\prime)} = \cos \theta^{(\prime)}, \sin \theta^{(\prime)}$ with

$$\theta = \arctan \frac{v_1 v_2 (\lambda_4 + \lambda_7)}{m_2^2 + v_2^2 \lambda_2 - m_1^2 - v_1^2 \lambda_1}, \quad \theta' = \arctan \frac{v_3 v_4 (\lambda_{12} - 2\alpha)}{v_4^2 \lambda_4 - v_3^2 \lambda_3 + \alpha(v_4^2 - v_3^2)}. \quad (11)$$

The mass matrix for the CP-odd Higgs fields is

$$M_{\text{odd}}^2 \approx \begin{pmatrix} m_1^2 & 0 & -v_4 \mu_1 & -v_3 \mu_1 \\ * & m_2^2 & -v_4 \mu_2 & -v_3 \mu_2 \\ * & * & \mu_1 v_1 v_3^{-1} v_4 + \mu_2 v_2 v_3^{-1} v_4 & -v_1 \mu_1 + v_2 \mu_2 \\ * & * & * & \mu_1 v_1 v_4^{-1} v_3 + \mu_2 v_2 v_4^{-1} v_3 \end{pmatrix}, \quad (12)$$

which has two non-zero eigenvalues

$$M^2 = \frac{1}{2v_1 v_2 v_3 v_4} \left(v_2 \mu_1 [v_3^2 v_4^2 + v_1^2 (v_3^2 + v_4^2)] + v_1 \mu_2 [v_3^2 v_4^2 + v_2^2 (v_3^2 + v_4^2)] \pm \sqrt{\mathcal{Q} - \mathcal{P}} \right) \quad (13)$$

where

$$\mathcal{P} = 4 \frac{\mu_1 \mu_2}{v_1 v_2} \prod_i^4 v_i^2 \left[v_3^2 v_4^2 + v_2^2 (v_3^2 + v_4^2) + v_1^2 (4v_2^2 + v_3^2 + v_4^2) \right],$$

$$\mathcal{Q} = \left\{ v_2 [v_3^2 v_4^2 + v_1^2 (v_3^2 + v_4^2)] \mu_1 + v_1 [v_3^2 v_4^2 + v_2^2 (v_3^2 + v_4^2)] \mu_2 \right\}^2.$$

The other two are Goldstone bosons eaten by Z and Z' , separately.

Let's give some comments on the $Z - Z'$ mixing. Phenomenological constraints typically require the mixing angle to be less than $(1 \sim 2) \times 10^{-3}$ [26] and the mass of extra neutral gauge boson to be heavier than 860 GeV [27]. The multi-Higgs contributions to $Z - Z'$ mixing from both tree-level and one-loop level corrections are studied in Ref [25]. A suitable mass hierarchy and mixing between Z and Z' are maintained by setting $v_1, v_2 < 10$ GeV, $v_4 \sim 1$ TeV and $g \sim g_X$.

III. FERMION MASSES

Due to the flavor-dependent $U(1)_X$ symmetry, the Yukawa interaction of our model can be written as

$$-\mathcal{L}_{\text{Yukawa}} = +\bar{q}_L^u Y_{uu}^u \tilde{H}_1 u_R + \bar{q}_L^c Y_{cc}^u \tilde{H}_2 c_R + \bar{q}_L^t Y_{tt}^u \tilde{H}_3 t_R + \bar{q}_L^u Y_{ut}^u \tilde{H}_2 t_R + \bar{q}_L^c Y_{ct}^u \tilde{H}_1 t_R$$

$$\begin{aligned}
& + \overline{q}_L^u Y_{d\alpha}^d H_1 D_{R\alpha} + \overline{q}_L^c Y_{c\alpha}^d H_2 D_{R\alpha} + \overline{q}_L^t Y_{t\alpha}^d H_3 D_{R\alpha} \\
& + \overline{\ell}_L^\alpha Y_{\alpha e}^e H_1 e_R + \overline{\ell}_L^\alpha Y_{\alpha \mu}^e H_2 \mu_R + \overline{\ell}_L^\alpha Y_{\alpha \tau}^e H_3 \tau_R + \overline{\ell}_L^\alpha Y_{\alpha \beta}^\nu \tilde{H}_1 \nu_{R\beta} \\
& + \overline{\eta}_L^i Y_{ij}^\eta \Phi \eta_R + \overline{\xi}_L^i Y_{ij}^\xi \Phi^\dagger \xi_R + \overline{\ell}_L^\alpha Y_{\alpha k}^{mix} H_3 \eta_{Rk} + \overline{\ell}_L^\alpha Y_{\alpha k}^{mix'} H_3 \xi_{Rk} + \text{h.c.} \quad (14)
\end{aligned}$$

After $U(1)_X$ and electroweak symmetry spontaneously broken, we may get the mass matrix for the upper quarks and down quarks:

$$M_u = \begin{pmatrix} Y_{11}^u v_1 & 0 & Y_{13}^u v_2 \\ 0 & Y_{22}^u v_2 & Y_{23}^u v_1 \\ 0 & 0 & Y_{33}^u v_3 \end{pmatrix}, \quad M_d = \begin{pmatrix} Y_{11}^d v_1 & Y_{12}^d v_1 & Y_{13}^d v_1 \\ Y_{21}^d v_2 & Y_{22}^d v_2 & Y_{23}^d v_2 \\ Y_{31}^d v_3 & Y_{32}^d v_3 & Y_{33}^d v_3 \end{pmatrix}. \quad (15)$$

As we showed in the last section, v_i is hierarchical and we set $v_1 = 0.1$ GeV, $v_2 = 10$ GeV and $v_3 = 173$ GeV in our calculation. For simplification we may also set M_u , M_d to be nearly diagonal matrices using discrete flavor symmetry, such as Z_2^3 . Then v_i is only responsible for the origin of the i th generation quark masses. In that case all the Yukawa coupling constants, except that of top quark, are of $\mathcal{O}(10^{-2})$. Even for the most general case of Eq. 14, Yukawa coupling constant can be nearly at the same order. But we need to study constraint on the Yukawa couplings from electroweak precision measurements, which will be carried out in the next section.

The most general charged lepton mass matrix and Dirac neutrino mass matrix are

$$M_e = \begin{pmatrix} Y_{11}^e v_1 & Y_{12}^e v_1 & Y_{13}^e v_1 \\ Y_{21}^e v_2 & Y_{22}^e v_2 & Y_{23}^e v_2 \\ Y_{31}^e v_3 & Y_{32}^e v_3 & Y_{33}^e v_3 \end{pmatrix}, \quad M_D = v_1 \begin{pmatrix} Y_{11}^\nu & Y_{12}^\nu & Y_{13}^\nu \\ Y_{21}^\nu & Y_{22}^\nu & Y_{23}^\nu \\ Y_{31}^\nu & Y_{32}^\nu & Y_{33}^\nu \end{pmatrix}. \quad (16)$$

The charged lepton mass matrix is quite similar to that in the A_4 model [28, 29]. We set it to be diagonal using $Z_2 \times Z_2 \times Z_2$ flavor symmetry, which is explicitly broken by neutrino Yukawa interactions. In this case Y_{ii}^e is of order $\mathcal{O}(10^{-2})$. The Dirac neutrino mass matrix is proportional to v_1 , thus it can be at the MeV scale without requiring relatively small neutrino Yukawa couplings. The right handed neutrino masses may come from the effective operator $\alpha \Lambda^{-1} \Phi^2 \overline{\nu}_R^C \nu_R + \text{h.c.}$. Integrating out heavy neutrinos, we derive the mass matrix of active neutrinos: $M_\nu = v_1^2 Y^\nu M_R^{-1} Y^{\nu T}$. Setting $\mathcal{O}(Y^\nu) \sim 10^{-2}$ and $M_R \sim 100$ GeV, we derive electron-volt scale active neutrino masses.

η and ξ get masses after the $U(1)_X$ symmetry spontaneously broken. Besides they mix with the charged leptons through the Yukawa interactions. To be consistent with the EW precision measurements, we assume the mixing is relatively small. ψ_L may get the mass in

the same way as that of right-handed neutrinos. It can be stable particle with the help of Z_2 flavor symmetry, thus it can be dark matter candidate. It's phenomenology will be studied in section V.

IV. CONSTRAINTS

There are two major constraints on any extension of the Higgs sector of the SM.: the ρ parameter and the flavor changing neutral currents(FCNC). Notice that in a model with only Higgs doublet, the tree level of $\rho = 1$ is automatic without adjustment to any parameters in the model. For our model ρ is maintained as the constraint on the $Z - Z'$ mixing is fulfilled. Our model doesn't obey the theorem called Natural Flavor Conservation by Glashow and Weinberg, such that there are tree level FCNC's mediated by the Higgs boson. In the basis where M_u is diagonalized, M_D can be written as

$$M_d = \mathcal{U}_{CKM} \cdot \hat{D} \cdot U_R^\dagger \Rightarrow Y_D = \begin{pmatrix} v_1^{-1} & 0 & 0 \\ 0 & v_2^{-1} & 0 \\ 0 & 0 & v_3^{-1} \end{pmatrix} \mathcal{U}_{CKM} \hat{D} U_R^\dagger, \quad (17)$$

where $\hat{D} = \text{diag}\{m_d, m_s, m_b\}$. and \mathcal{U}_{CKM} is the CKM matrix. Then the flavor changing neutral current can be written as

$$\overline{(q_L^u \quad q_L^c \quad q_L^t)} \mathcal{U}_{CKM}^\dagger \text{Diag}\{v_1^{-1} H_1, v_2^{-1} H_2, v_3^{-1} H_3\} \mathcal{U}_{CKM} \hat{M}_D \begin{pmatrix} d_R \\ s_R \\ b_R \end{pmatrix} + \text{h.c.} \quad (18)$$

In this section, we consider various processes where FCNC may contribute significantly. Taking into account the experimental results of these processes, we may constrain the parameter spaces of the model.

A. $K - \bar{K}$ mixing

There are two well measured quantities related to $K - \bar{K}$ mixing: the mass difference and the CP violating observable. In this paper, we only focus on the contribution to the mass difference ΔM_K , which get its main contribution from the tree level exchange of h_i^0 (We assume CP-odd Higgs bosons being much heavier than CP-even ones, which dominate

the contributions to the $K - \bar{K}$ mixing). The relevant vertices can be read from Eq. 18:

$$\begin{cases} \overline{d}_L s_R h_i^0 & m_s v_i^{-1} \mathcal{U}_{i1}^* \mathcal{U}_{i2} , \\ \overline{s}_L d_R h_i^0 & m_d v_i^{-1} \mathcal{U}_{i2}^* \mathcal{U}_{i1} , \end{cases} \quad (19)$$

Thus the mass difference can be derived through the mass insertion method:

$$\Delta M_{12}^S = \sum_i \frac{f_K^2 m_K}{24 M_i^2} \left\{ \mathcal{A}_i^2 \left[-1 + \frac{11 m_K^2}{(m_s + m_d)^2} \right] + \mathcal{B}_i^2 \left[1 - \frac{m_K^2}{(m_s + m_d)^2} \right] \right\} , \quad (20)$$

where

$$\begin{aligned} \mathcal{A}_i &= \frac{1}{2} (m_s - m_d) v_i^{-1} \mathcal{U}_{i2}^* \mathcal{U}_{i1} , \\ \mathcal{B}_i &= \frac{1}{2} (m_s + m_d) v_i^{-1} \mathcal{U}_{i2}^* \mathcal{U}_{i1} . \end{aligned}$$

Using $f_K = 114$ MeV, $m_K = 497.6$ MeV and values of CKM matrix listed in PDG, We plot in the left panel of the Fig. 1 ΔM_K as the function of m_2 , the mass of the neutral component of the second Higgs doublet H_2 . In plotting the figure we set $v_1 = 0.1$ GeV, $v_2 = 10$ GeV, $v_3 = 173$ GeV as well as $m_1 = 20m_2$, which is natural because v_i ($i = 1, 2$) is inverse proportional to the m_i^2 . The horizontal line in the figure represents the experimental value. To fulfill the experimental constraint, m_2 should be no smaller than 8.66 TeV in our model. This value might be accessible at the future LHC.

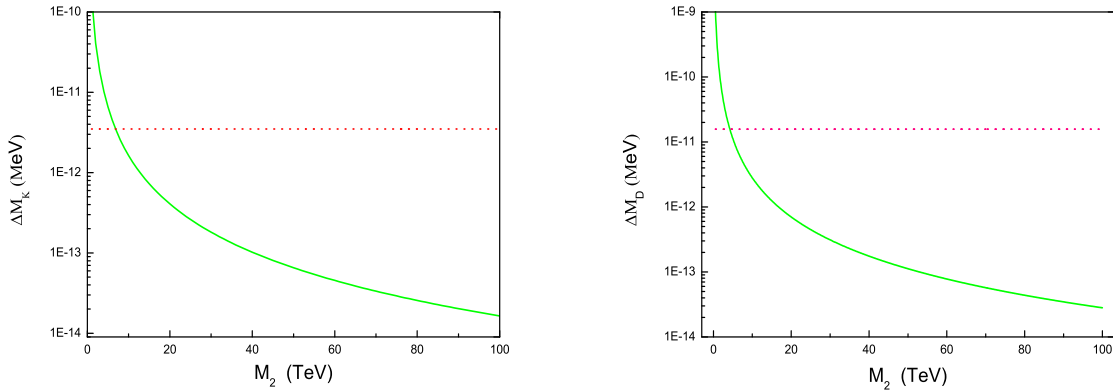


FIG. 1: ΔM_K (the left panel of the figure) and ΔM_D (the right panel of the figure) as the function of m_2 the mass eigenvalue of the h_2^0 .

B. $D - \bar{D}$ mixing

The $D - \bar{D}$ mixing in our model is a little different form that of $K - \bar{K}$ mixing. The contributions to the $D - \bar{D}$ mixing come from box diagrams, which include the SM W boson diagram, the two Higgs diagrams and the mixed diagrams. We assume the two Higgs diagrams dominant the contribution. The following are relevant vertices :

$$\begin{cases} \overline{c}_L d_R h_i^+ : & m_d v_i^{-1} \mathcal{U}_{i2}^* \mathcal{U}_{i1} , \\ \overline{c}_L s_R h_i^+ : & m_s v_i^{-1} \mathcal{U}_{i2}^* \mathcal{U}_{i2} , \\ \overline{c}_L b_R h_i^+ : & m_b v_i^{-1} \mathcal{U}_{i2}^* \mathcal{U}_{i3} , \end{cases} \quad \begin{cases} \overline{u}_L d_R h_i^+ : & m_d v_i^{-1} \mathcal{U}_{i1}^* \mathcal{U}_{i1} , \\ \overline{u}_L s_R h_i^+ : & m_s v_i^{-1} \mathcal{U}_{i1}^* \mathcal{U}_{i2} , \\ \overline{u}_L b_R h_i^+ : & m_b v_i^{-1} \mathcal{U}_{i1}^* \mathcal{U}_{i3} , \end{cases} \quad (21)$$

Then we have

$$M_{12}^D = \frac{1}{384\pi^2} \Lambda^2 f_D^2 m_D \sum_m \sum_n y_m y_n \sum_{ij} \mathcal{Y}_{um}^i \mathcal{Y}_{cm}^{j*} \mathcal{Y}_{un}^j \mathcal{Y}_{cn}^{i*} \mathcal{I}(y_m, y_n, y_i, y_j) , \quad (22)$$

where $y_\alpha, y_\beta = m_{\alpha,\beta}^2/\Lambda^2$ and $\mathcal{Y}_{mn}^i = v_i^{-1} \mathcal{U}_{im}^* \mathcal{U}_{in}$. The explicit expression of integration $\mathcal{I}(a, b, c, d)$ can be found in Ref. [18].

Using $f_D = 170$ MeV and $M_D = 1864$ MeV, we plotting in the right panel of Fig. 1 ΔM_D as a function of m_2 . Our parameter settings are the same as that of the $K - \bar{K}$ mixing. the horizontal line in the figure represent the experimental value. We can read from the figure that the data of $D - \bar{D}$ mixing constraints the mass of h_2^+ to be no smaller than 4.2 TeV.

C. $B - \bar{B}$ mixing

The mass difference in the neutral B meson system has been well measured by the D0 Collaboration and the CDF Collaboration at the Fermilab Tevatron. Similar to that of $K - \bar{K}$ mixing, there are also tree-level contributions to the ΔM_{B_α} . The following are relevant vertices that might lead to $B_\alpha - \bar{B}_\alpha$ mixing:

$$\begin{cases} \overline{d}_L b_R h_i^0 & m_b v_i^{-1} \mathcal{U}_{i1}^* \mathcal{U}_{i3} , \\ \overline{b}_L d_R h_i^0 & m_d v_i^{-1} \mathcal{U}_{i3}^* \mathcal{U}_{i1} , \end{cases} \quad \begin{cases} \overline{s}_L b_R h_i^0 & m_b v_i^{-1} \mathcal{U}_{i2}^* \mathcal{U}_{i3} , \\ \overline{b}_L s_R h_i^0 & m_s v_i^{-1} \mathcal{U}_{i3}^* \mathcal{U}_{i2} , \end{cases} \quad (23)$$

Direct calculation gives

$$\Delta M_{12}^{B_\alpha} = \sum_i \frac{f_B^2 m_{B_\alpha}}{24 M_i^2} \left\{ \mathcal{C}_{\alpha i}^2 \left[-1 + \frac{11 m_K^2}{(m_s + m_d^2)} \right] + \mathcal{D}_{\alpha i}^2 \left[1 - \frac{m_K^2}{(m_s + m_d)^2} \right] \right\} , \quad (24)$$

where

$$\begin{aligned} \mathcal{C}_{\alpha i} &= \frac{1}{2} (m_b - m_\alpha) v_i^{-1} \mathcal{U}_{i3}^* \mathcal{U}_{j\alpha} , \\ \mathcal{D}_{\alpha i} &= \frac{1}{2} (m_b + m_\alpha) v_i^{-1} \mathcal{U}_{i3}^* \mathcal{U}_{j\alpha} , \end{aligned}$$

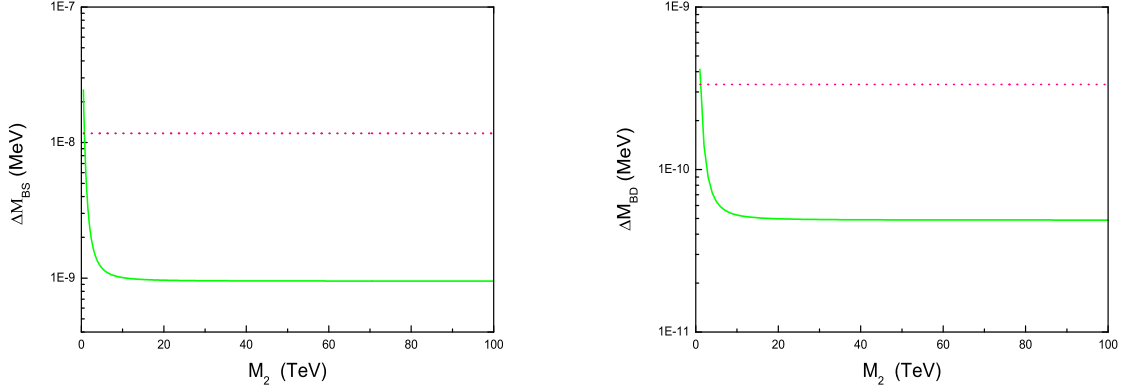


FIG. 2: ΔM_{BS} (the left panel of the figure) and ΔM_{BD} as the function of m_2 the mass eigenvalue of the ϕ_2^0 .

and $m_{B_s} = 5367.5$ MeV, $m_{B_0} = 5279.4$ MeV. Using the same input as that of the $K - \bar{K}$ mixing case, we plot in the left panel of Fig. 2 ΔM_{B_0} and in the right panel ΔM_{B_s} as the function of m_2 , where the horizontal lines in both cases represent the corresponding experimental data. Our results show that ΔM_{B_α} is not so sensitive to m_2 , which is because $H_{2s'}$ contribution is heavily suppressed by the CKM. Our numerical results shows that m_2 should be no smaller than 0.8 TeV.

D. $\mu \rightarrow e\gamma$

Now we come the lepton sector and discuss constraint on the model from lepton flavor violating decays. Among the current available experimental data, $\mu \rightarrow e\gamma$ gives the strongest constraint. We assume the Yukawa matrix for the charged leptons is diagonal such that the only relevant Yukawa interactions are $\ell_L Y^\nu \tilde{H}_1 N_R + \text{h.c.}$. Their contribution to the $\mu \rightarrow e\gamma$ can be written as

$$BR(\mu \rightarrow e + \gamma) = \frac{3e^2}{64\pi^2 G_F^2} |\mathcal{F}|^2 \left(1 - \frac{m_e^2}{m_\mu^2}\right)^3, \quad (25)$$

with

$$\mathcal{F} = \frac{Y_{ei}^\nu Y_{\mu i}^{\nu*}}{12(m_1'^2 - m_{Ni}^2)} \left\{ -2 + \frac{9m_1'^2}{m_1'^2 - m_{Ni}^2} - 6 \left(\frac{m_1'^2}{m_1'^2 - m_{Ni}^2} \right)^2 + \frac{6m_{Ni}^4 m_1'^2}{(m_1'^2 - m_{Ni}^2)^3} \ln \left(\frac{m_1'^2}{m_{Ni}^2} \right) \right\} \quad (26)$$

where m'_1 is the mass eigenvalue of h_1^\pm and m_{Ni} is the mass eigenvalues of right handed neutrinos. In deriving the upper results we have assumed $m_{Ni} < m'_1$.

The current experimental upper bounds for the $BR(\mu \rightarrow e\gamma)$ is 1.2×10^{-11} . By assuming $m'_1 \sim 4.5$ TeV and $m_{Ni} \sim 500$ GeV, we can get the upper bound for the $Y_{ei}Y_{\mu i}^*$ which is about of order 1, i.e., there are no severe constraint on the neutrino Yukawa couplings from lepton flavor violations.

V. DARK MATTER

In our model the neutral fermions ψ_L (introduced to cancel the anomalies of N_R) is stable and thus can be dark matter candidate. Its relic density can be written as

$$\Omega h^2 \simeq \frac{1.07 \times 10^9 \text{ GeV}^{-1}}{M_{Pl}} \frac{x_f}{\sqrt{g_*}} \left(\frac{19M_\chi^2 g_\chi^4}{4\pi [(4M_\chi^2 - M_{Z'}^2)^2 + M_{Z'}^2 \Gamma_{Z'}^2]} x^{-1} \right)^{-1} \quad (27)$$

where h is the Hubble constant in units of $100 \text{ km/s} \cdot \text{Mpc}$, $M_{Pl} = 1.22 \times 10^{19} \text{ GeV}$ is the Planck mass, g_* accounts the number of relativistic degrees of freedom at the freeze-out temperature and $M_{Z'}$ is the mass of Z' with $\Gamma_{Z'}$ its decay width. We set x_f equals to 20 in our calculation, a typical value at the freeze-out for weakly interacting particles.

The elastic scattering cross section of Dark matter off the nucleon can be written as

$$\sigma_n^{\text{SD}}(\chi + n \rightarrow \chi + n) = \frac{6}{\pi} \left(\frac{M_n M_\chi}{M_n + M_\chi} \right)^2 \left(\sum_{q=u,d,s} d_q \Delta q^{(n)} \right)^2 \quad (28)$$

We follow the DARKSUSY[30] and use the following inputs for the spin-dependent calculations:

$$\begin{aligned} \Delta_u^p &= +0.77, & \Delta_d^p &= -0.40, & \Delta_s^p &= -0.12, \\ \Delta_u^n &= -0.40, & \Delta_d^n &= +0.77, & \Delta_s^n &= -0.12. \end{aligned} \quad (29)$$

For our model, the coefficient d_q can be written as

$$d_q = \frac{1}{4} a_q g'^2 M_{z'}^{-2}, \quad (30)$$

where a_q is the hypercharge of quarks under the new $U(1)$ gauge symmetry.

The cosmological experiments have precisely measured the relic density of the non-baryonic cold dark matter: $\Omega_D h^2 = 0.1123 \pm 0.0035$ [31]. Taking this result into Eq. 27, we

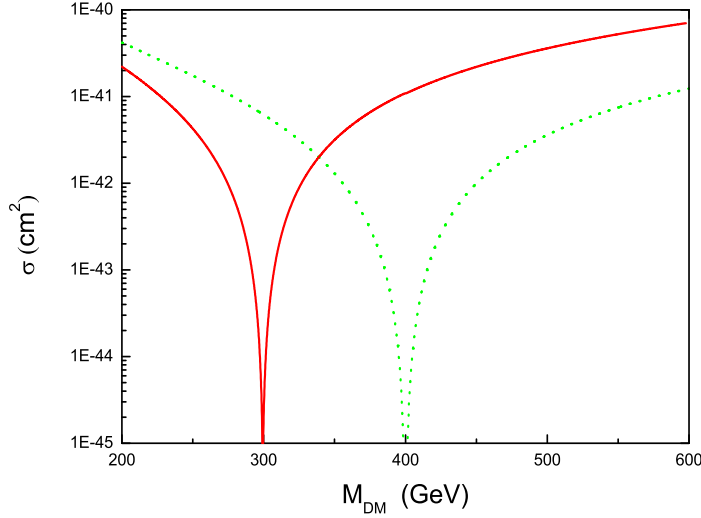


FIG. 3: $\sigma(\chi + n \rightarrow \chi + n)$ as function of dark matter mass M_{DM} constrained dark matter relic density.

may derive g_X as implicit function of M_{DM} and $M_{Z'}$. Then one free parameter is reduced. We plot in Fig. 3 $\sigma(\chi n \rightarrow \chi n)$ as the function of the mass of the dark matter constrained by the dark matter relic density. The solid and dotted lines correspond to $M_{Z'} = 600$ and 800 GeV, separately. The Xenon-100 [32] gives the strongest constraint on the dark matter-nucleon scattering cross section in the region, which is about $[1 \times 10^{-44}, 4 \times 10^{-44}]$. It constrains M_{DM} lying near $1/2 M_{Z'}$ for our model, around which all the experimental constraints may be fulfilled.

VI. CONCLUSION

In this paper, we proposed a possible solution to the fermion mass hierarchy problem by fitting the type-II seesaw mechanism into the Higgs doublet sector. We extended the Standard Model with two extra Higgs doublets as well as a spontaneously broken $U_X(1)$ gauge symmetry. The VEVs of Higgs doublets are normal hierarchal due to the $U(1)_X$ symmetry. In our model all the Yukawa couplings of quarks and leptons except that of top quark, are of order $\mathcal{O}(10^{-2})$. Constraints on the model from meson mixings, lepton flavor violations as well as dark matter direct detection were studied. The masses of new Higgs

fields can be several TeV, the collider signatures of which are important but beyond the scope of this paper will be shown in somewhere else.

Acknowledgments

The author is indebted to Prof. M. Ramsey-Musolf for his hospitality at the UW and Prof. X. G. He for his hospitality at the SJTU.

Appendix A: Diagonalization of 4×4 Higgs mass matrix

The CP-even Higgs matrix can only be block diagonalized. We first write it as

$$M_{\text{CP-even}}^2 = \begin{pmatrix} \mathcal{Z} & \mathcal{T} \\ \mathcal{T}^T & \mathcal{Z}' \end{pmatrix} \quad (\text{A1})$$

where \mathcal{Z} , \mathcal{T} and \mathcal{Z}' are 2×2 sub-matrix with

$$\mathcal{Z} = \begin{pmatrix} m_1^2 + v_1^2 \lambda_1 & \frac{1}{2} v_1 v_2 (\lambda_4 + \lambda_7) \\ * & m_2^2 + v_2^2 \lambda_2 \end{pmatrix}, \quad (\text{A2})$$

$$\mathcal{T} = \begin{pmatrix} \frac{1}{2} v_1 v_3 (\lambda_5 + \lambda_8) - \mu_1 v_4 & \frac{1}{2} v_1 v_4 \lambda_{10} - v_3 \mu_1 \\ \frac{1}{2} v_1 v_3 (\lambda_6 + \lambda_9) - \mu_2 v_4 & \frac{1}{2} v_1 v_4 \lambda_{11} - v_3 \mu_2 \end{pmatrix}. \quad (\text{A3})$$

-
- [1] K. Nakamura *et al.*, (Particle Data Group), J. Phys. G **37**, 075021 (2010).
 - [2] P. Minkowski, Phys. Lett. B **67**, 421 (1977); T. Yanagida, in *Workshop on Unified Theories*, KEK report 79-18 p.95 (1979); M. Gell-Mann, P. Ramond, R. Slansky, in *Supergravity* (North Holland, Amsterdam, 1979) eds. P. van Nieuwenhuizen, D. Freedman, p.315; S. L. Glashow, in *1979 Cargese Summer Institute on Quarks and Leptons* (Plenum Press, New York, 1980) eds. M. Levy, J.-L. Basdevant, D. Speiser, J. Weyers, R. Gastmans and M. Jacobs, p.687; R. Barbieri, D. V. Nanopoulos, G. Morchio and F. Strocchi, Phys. Lett. B **90**, 91 (1980); R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980); G. Lazarides, Q. Shafi and C. Wetterich, Nucl. Phys. B **181**, 287 (1981).
 - [3] W. Konetschny and W. Kummer, Phys. Lett. B **70**, 433 (1977); T. P. Cheng and L. F. Li, Phys. Rev. D **22**, 2860 (1980); G. Lazarides, Q. Shafi and C. Wetterich, Nucl. Phys. B **181**,

- 287 (1981); J. Schechter and J. W. F. Valle, Phys. Rev. D **22**, 2227 (1980); R. N. Mohapatra and G. Senjanovic, Phys. Rev. D **23**, 165 (1981).
- [4] R. Foot, H. Lew, X. G. He and G. C. Joshi, Z. Phys. C **44**, 441 (1989).
 - [5] C. D. Froggatt and Nielsen, Nucl. Phys. B **147**, 277(1979).
 - [6] K. R. Dienes, E. Dudas and T. Gherghetta, Phys. LettB **436**, 55 (1998).
 - [7] I. Gogoladze, C. A. Lee, T. Li and Q. Shafi, Phys. Rev. D **78**, 015024 (2008).
 - [8] H. Frtzsch and Z. Z. Xing, Prog. Par. Nucl. Phys. **45**, 1 (2000).
 - [9] Y. Buchmuller and T. Yanagida, Phys. Lett. B **445**, 399 (1999).
 - [10] Y. Nir, Phys. Lett. B **354**, 107 (1995).
 - [11] J. J. Heckman and C. Vafa, Nucl. Phys. B **837**, 137 (2010).
 - [12] F. Bazzocchi, M. Frigerio and S. Morisi, Phys. Rev. D **78**, 116018 (2008).
 - [13] K. Koshioka, Mod. Phys. Lett. A **15**, 29 (2000).
 - [14] S. Davidson, G. Isidori and S. Uhlig, Phys. Lett. B **63**, 73 (2008).
 - [15] G. J. Ding, Phys. Rev. D **78**, 036011 (2008).
 - [16] C. D. Froggatt, G. Lowe and H. B. Nielsen, Nucl. Phys. B **414**, 579 (1994).
 - [17] F. Feruglio and Y. Lin, Nucl. Phys. B **800**, 77 (2008).
 - [18] Y. Grossman, Nucl. Phys. B **426**, 355 (1994).
 - [19] R. A. Porto and A. Zee, Phys. Lett. B **666**, 491 (2008); Phys. Rev. D **79**, 013003 (2009).
 - [20] E. Ma, Phys. Rev. Lett. **86** 2502 (2001); Phys. Lett. B **516**, 165 (2001).
 - [21] S. M. Davidson, H. E. Logan, Phys. Rev. D **80**, 095008 (2009); T. Morozumi, H. Takata and K. Tamai, arXiv: 1009.1026[hep-ph].
 - [22] F. Josse-Michaux and E. Molinaro, arXiv:1109.0482[hep-ph].
 - [23] N. Haba and O. Seto, arXiv:1106.5353[hep-ph]; Prog. Theor. Phys. **125**, 1155 (2011); N. Haba and K. Tsumura, JHEP **1106**, 068 (2011); N. Haba and M. Hirotsu, Eur. Phys. J. C **69**, 481 (2010).
 - [24] W. Grimus and L. Lavoura, Phys. Lett. B **687**, 188 (2010).
 - [25] W. Chao and M. Ramsey-Musolf, to appear.
 - [26] P. Abreu, *et al.*, (DELPHI Collaboration), Phys. Lett. B **485**, 45 (2000); R. Barate *et al.*, (ALEPH Collaboration), Eur. Phys. J. C. **12**, 183 (2000); J. Erler, P. Langacker, S. Munir and E. R. Pena, arXiv: 0906.2345.
 - [27] J. F. Grivaz, Int. J. Mod. Phys. A **23**, 3849 (2008) and reference therein.

- [28] K. S. Babu, E. Ma and J. W. F. Valle, Phys. Lett. B **552**, 207 (2003).
- [29] X. G. He, Y. Y. Keum and R. R. Volkas, JHEP **0604**, 039 (2006).
- [30] P. Gondolo, J. Edsjo, P. Ullio, L. Bergstrom, M. Schelke and E. A. Baltz, JCAP **0407**, 008 (2004).
- [31] E. Komatsu, *et al.*, arXiv: 1001.4538[astro-ph.CO]
- [32] E. Aprile *et al.*, XENON 100 Collaboration, Phys. Rev. Lett. **107**, 131302, (2011).